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Answers Key:

5 மதிப்பெண் வினாக்கள்

1. $\int \frac{3x+2}{(x-2)^2(x-3)}dx = \int \left[\frac{11}{(x-2)} - \frac{8}{(x-2)^2} + \frac{11}{(x-3)}\right]dx$
$= 11\int \frac{dx}{(x-2)} - 8\int \frac{dx}{(x-2)^2} + 11\int \frac{dx}{(x-3)}$
$= 11\log|x-2| + \frac{8}{x-2} + 11\log|x-3| + c$
$= 11\log\left|\frac{x-3}{x-2}\right| + \frac{8}{x-2} + c$
பகுதி பின்னமாக்கல் முறையில்,
$\frac{3x+2}{(x-2)^2(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)}$
$\Rightarrow \frac{3x+2}{(x-2)^2(x-3)} = \frac{8}{(x-2)^2} + \frac{11}{(x-3)}$
2. $\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}dx$
$\int \frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{A}{x-1} + \frac{BX+C}{x^2+5}$
$3x^2 - 2x + 5 = A(x^2+5) + (Bx+C) + (x-1)$ .....(1)
x=1 எனில்
3-2 +5 =A (6)
6A =6 $\Rightarrow$A = 1
x=0 எனில் 5 =5A -C $\Rightarrow$C =0
(1) -ல் x^2 -ன் கெழுக்களை சமப்படுத்த
$3 = A + B \Rightarrow B = 2$
$\frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{1}{x-1} + \frac{2x}{x^2+5}$
$\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}dx = \int \frac{dx}{x-1} + \int \frac{2x}{x^2+5}dx$
$= \log|(x-1)| + \log|x^2+5| + c$
$= \log|(x-1)(x^2+5)| + c$
$= \log|x^3 - x^2 + 5x - 5| + c$
மாற்று முறை
$\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}dx = \int \frac{3x^2-2x+5}{x^3-x^2+5x-5^4}$
$t = x^3 - x^2 + 5x - 5$
$dt = (3x^2 - 2x + 5)dx$
$\int \frac{dt}{t} = \log|t| + c$
$= \log|x^3 - x^2 + 5x - 5| + c$
3. $\int (e^x(1+x)\log(xe^x))dx = \int \log t dt$

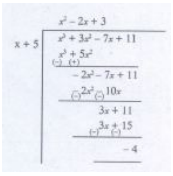
$t = xe^x$
$dt = (xe^x + e^x)dx$
$= e^x(x+1)dx$
$I = \int \log t dt$
$\int dv = \int dt$
$u = \log t$
$du = \frac{1}{t}dt$
v=t

$\int udv = uv - \int vdu$
$\int \log t dt = t\log t - \int t\frac{1}{t}dt = t\log t - t + c$
$\int e^x(1+x)\log(xe^x)$
$= xe^x\log(xe^x) - xe^x + c$
$= xe^x[\log(xe^x) - 1] + c$
4. $\int e^{3x}\left[\frac{3x-1}{9x^2}\right]dx$
$= \int e^{3x}\left(\frac{3x}{9x^2} - \frac{1}{9x^2}\right)dx$
$= \int e^{3x}\left[\frac{3}{9x} - \frac{1}{9x^2}\right]dx$
$f(x) = \frac{1}{9x}$ மற்றும் a = 3
$\Rightarrow f(x) = \frac{1}{9x}.x^{-1}$
$f'(x) = \frac{1}{9}.(-1)x^{-1-1}$
$\Rightarrow f'(x) = \frac{-1}{9}x^{-2} = \frac{-1}{9x^2}$
$\therefore I = \int e^{3x}(3.f(x) + f'(x))dx$
$= e^{3x}f(x) + c$
$\left[\because \int e^{ax}[af(x) + f'(x)]dx = e^{ax}f(x) + c\right]$
$= e^{3x}\frac{1}{9x} + c = \frac{e^{3x}}{9x} + c$
5. $\int_2^3 \frac{x^4+1}{x^2}dx = \int_2^3 (x^2 + x^{-2})dx$
$= \left[\frac{x^3}{3} - \frac{1}{x}\right]_2^3$
$= (9 - \frac{1}{3})\left(\frac{8}{3} - \frac{1}{2}\right) = \frac{13}{2}$

3 மதிப்பெண் வினாக்கள்

6.

$\int \frac{x^3+3x^2-7x+11}{x+5}dx$



$x^3+3x^2-7x+11=(x+5)\left(x^2-2x+3\right)-4$

$\int \frac{x^3+3x^2-7x+11}{x+5}dx$

$=\int\left(x^2-2x+3\right)dx-4\int\frac{dx}{x+5}$

$=\frac{x^3}{3}-\frac{2x^2}{2}+3x-4\log|x+5|+c$

$=\frac{x^3}{3}-x^2+3x-4\log|x+5|+c$

7. $f'(x)=e^x$

$f(x)=\int e^x dx$

$=e^x+c$

$f(0)=2$

$2=e^0+c\Rightarrow c=2-1=1$

f(x)=e^x+1

8. $\int\left(\frac{\cos 2x+2\sin^2 x}{\cos^2 x}\right)dx=\int\frac{1-2\sin^2 x+2\sin^2 x}{\cos^2 x}dx$

$=\int\frac{dx}{\cos^2 x}=\int\sec^2 x dx=\tan x+c$

9. $\int\left(x^3e^{3x}\right)dx$

$u=x^3$

$u'=3x^2$

$u''=6x$

$u''=6$

$\int dv=\int e^{3x}dx$

$v=\frac{e^{3x}}{3}$

$v_1=\frac{e^{3x}}{9}$

$v_2=\frac{e^{3x}}{27}$

$v_3=\frac{e^{3x}}{81}$

$\int udv=uv-u'v_1^-+u''v_2-u''v_3+\ldots$

$\int x^3e^{3x}dx=\frac{x^3e^{5x}}{3}-\frac{3x^2e^{3x}}{9}+\frac{6xe^{3x}}{27}-\frac{6e^{3x}}{81}+c$

$=e^{3x}\left[\frac{x^3}{3}-\frac{x^2}{3}+\frac{2x}{9}-\frac{2}{27}\right]+c$

10. $\int\frac{dx}{x+\sqrt{x^2+1}}$

$\int\frac{1}{x+\sqrt{x^2-1}}\times\frac{x-\sqrt{x^2-1}}{x-\sqrt{x^2-1}}dx$

$=\int\frac{\left(x-\sqrt{x^2-1}\right)dx}{x^2-(x^2-1)}$

$\left[\because(a+b)(a-b)=a^2-b^2\right]$

$=\int\frac{\left(x-\sqrt{x^2-1}\right)dx}{x^2-x^2-1}$

$\int\left(x-\sqrt{x^2-1}\right)dx$

$=\int x\quad dx-\int\sqrt{x^2-1}dx$

$=\frac{x^2}{2}-\left[\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\log\left|x+\sqrt{x^2-}\right|+c\right]$

$\left[\because\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$=\frac{x^2}{2}-\frac{x}{2}\sqrt{x^2-1}+\frac{1}{2}log\left|x+\sqrt{x^2-1}\right|+c$

11. இங்கு, $\frac{dy}{dx}=\frac{2x}{5x^2+1}$

$\therefore y=\int_0^1\frac{2x}{5x^2+1}dx$

$=\frac{1}{5}\int_0^1\frac{10x}{5x^2+1}dx$

$=\frac{1}{5}\left[\log(5x^2+1)\right]_0^1=\frac{1}{5}[\log 6-\log 1]$

$=\frac{1}{5}\log 6$

12. $\int_0^{\frac{\pi}{2}}\sqrt{1+\cos x}dx$

cos 2x = 2 $\cos^2$ x-1

$\Rightarrow$ 1+cos 2x = 2 $\cos^2$x

$\Rightarrow$ 1+cos 2x = 2 $\cos^2\frac{x}{2}$

$\int_0^{\frac{\pi}{2}}\sqrt{1+\cos x}dx=\int_0^{\frac{\pi}{2}}\sqrt{2\cos^2\frac{x}{2}}dx$

$=\sqrt{2}\int_0^{\frac{\pi}{2}}\cos\frac{x}{2}dx$

$=\sqrt{2}\left(\frac{\sin\frac{x}{2}}{\frac{1}{2}}\right)_0^{\frac{\pi}{2}}$

$=2\sqrt{2}\left(\sin\frac{\pi}{2(2)}-\sin\frac{0}{2}\right)$

$=2\sqrt{2}\left(\sin\frac{\pi}{4}-\sin 0\right)$

$=2\sqrt{2}\left[\frac{1}{\sqrt{2}}-0\right]$

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$= \frac{2\sqrt{2}}{\sqrt{2}} = 2$

13. $\int_1^2 \frac{x-1}{x^2} dx$

$= \int_1^2 \left(\frac{x}{x^2} - \frac{1}{x^2}\right) dx$

$= \int_1^2 \left(\frac{1}{x} - x^{-2}\right) dx$

$= \left(\log x - \frac{x^{-2+1}}{-2+1}\right)_1^2$

$= \left(\log x - \frac{x^{-1}}{-1}\right)_1^2 = \left(\log x - \frac{1}{x}\right)_1^2$

$= \left(\log 2 + \frac{1}{2}\right) - \left(\log 1 + \frac{1}{1}\right)$

$= \log 2 + \frac{1}{2} - 1$

$= \log 2 - \frac{1}{2}$

$= \frac{2log2-1}{2}$

$= \frac{1}{2}(2\log 2 - 1)$

14.

$I = \int \log\left(x - \sqrt{x^2-1}\right) dx$

$u = \log\left(x - \sqrt{x^2-1}\right)$

dv = dx

v = x

$du = \frac{1}{x - \sqrt{x^2-1}} \left[\frac{d}{dx}\left(x - \sqrt{x^2-1}\right)\right] dx$

$= \frac{1}{x - \sqrt{x^2-1}} \left(1 - \frac{1}{2}\left(x^2-1\right)^{\frac{1}{2}-1} (2x)\right) dx$

$= \frac{1}{x - \sqrt{x^2-1}} \left(1 - \frac{x}{\sqrt{x^2-1}}\right) dx$

$= \frac{1}{x - \sqrt{x^2-1}} \left(\frac{\sqrt{x^2-1} - x}{\sqrt{x^2-1}}\right) dx$

$du = \frac{-1}{\sqrt{x^2-1}} dx$

$I = \int u dv == uv - \int v du$

$= \int \log(x - \sqrt{x^2-1}) dx$

$= x \log\left(x - \sqrt{x^2-1}\right) - \int \left(\frac{x dx}{\sqrt{x^2-1}} dx\right)$

$= I = x \log\left(x - \sqrt{x^2-1}\right) + I_1$

$I_1 = \int \frac{x}{\sqrt{x^2-1}} dx$

t =x^2-1

$dt = 2x dx \Rightarrow \frac{dt}{2} = x dx$

$I_1 = \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \frac{1}{2} \int t^{-\frac{1}{2}} dt$

$= \frac{1}{2}\left(\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) = \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}}$

$= \sqrt{t} = \sqrt{x^2-1}$

$\left[\because t = x^2 - 1\right]$

$I = x \log\left(x - \sqrt{x^2-1}\right) + \sqrt{x^2-1} + c$

15. $I = \int_{-1}^{1} x^2 e^{-2x} dx$

u = x^2 dv = e^{-2x}

u^1 = 2x $v = \frac{e^{-2x}}{-2} = \frac{-e^{-2x}}{2}$

u^4 = 2 $v_1 = +\frac{e^{-2x}}{4}$

$v_2 = -\frac{e^{-2x}}{8}$

I = uv = $u^1v_1 + u^4v_2$

$= \left[\left(\frac{x^2 e^{-2x}}{-2}\right) - 2x\left(\frac{e^{-2x}}{4}\right) + 2\left(\frac{e^{-2x}}{-8}\right)\right]_{-1}^{1}$

$= -\left[\frac{e^{-2}}{2} - \frac{e^{-2}}{2} - \frac{e^{-2}}{4}\right] - \left[\frac{e^2}{2} - \frac{e^{+2}}{2} - \frac{e^2}{4}\right]$

$\left(e^{-2x}\left[\frac{-x^2}{2} - \frac{x}{2} - \frac{1}{4}\right]\right)$

$= e^{-2}\left(-\frac{5}{4}\right) - e^2\left(-\frac{1}{4}\right)$

$= \frac{-5}{4}e^{-2} + \frac{1}{4}e^2$

$= \frac{1}{4}\left(e^2 - \frac{5}{e^2}\right) = \frac{1}{4}\left(\frac{e^4-5}{e^2}\right)$

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2 மதிப்பெண் வினாக்கள்

16.

$\int \frac{ax^2+bx+c}{\sqrt{x}}dx = \int \left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}} + cx^{-\frac{1}{2}}\right)dx$

$= a\int x^{\frac{5}{3}}dx + b\int x^{\frac{1}{3}} + c\int x^{-\frac{1}{2}}dx$

$= \frac{2ax^{\frac{5}{3}}}{5} + \frac{2bx^{\frac{3}{2}}}{3} + 2cx^{\frac{1}{2}} + k.$

17. $\int \sqrt{3x+5}dx = \int (3x+5)^{\frac{1}{2}}dx$

$= \frac{(3x+5)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)} + c$

$= \frac{(3x+5)^{\frac{3}{2}}}{9/2} + c$

$= \frac{2}{9}(3x+5)^{\frac{3}{2}} + c$

18. $\int \sin^2 x dx = \int \frac{1}{2}(1-\cos 2x)dx$

$= \frac{1}{2}\left[\int dx - \int \cos 2x dx\right]$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + c$

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cos2x = 1- 2sin^2 x

$\therefore \sin^2 x = \frac{1}{2}(1-\cos 2x)$

19. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x}dx = \int (cosec^2 x - \sec^2 x)dx$

= −cot x − tan x + c

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$\frac{\cos 2x}{\sin^2 x \cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x}$

$= \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}$

=cosec^{2}x-sec^{2}x

20. $\int \sqrt{1+\sin 2x dx} = \int \sqrt{(\sin x + \cos x)^2 dx}$

$= \int (\sin x + \cos x)dx$

=−cos x + sin x + c

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1+ sin2x = sin^2 x + cos^2 x + 2sin x cos x

= (sin x + cos x)^2

21. $\int (4x-2)\sqrt{x^2+x+1}\,dx$

$= 2\int (2x+1)\sqrt{x^2+x+1}\,dx$

$= 2\int f'(x) f(x)^{1/2}\,dx$

$= 2\frac{[f(x)]^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + c$

f(x) =x^2 +x +1

f′(x) =(2x+1)

22. $\int \frac{1}{2x^2-9}dx$

$= \frac{1}{2}\int \frac{dx}{x^2-\frac{9}{2}}$

$= \frac{1}{2}\int \frac{dx}{x^2-\left(\frac{3}{\sqrt{2}}\right)^2}$

$= \frac{1}{2} \times \frac{1}{2\left(\frac{3}{\sqrt{2}}\right)}\log\left|\frac{x-\frac{3}{\sqrt{2}}}{x+\frac{3}{\sqrt{2}}}\right| + c$

$\left[\because \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|\right] + c$

$= \frac{1}{\frac{3\sqrt{2}}{\sqrt{2}}}\log\left|\frac{\sqrt{2}x-3}{\sqrt{2}x+3}\right| + c$

$= \frac{1}{6\times\frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}}}\log\left|\frac{\sqrt{2}x-3}{\sqrt{2}x+3}\right| + c$

$= \frac{1}{6\times\sqrt{2}}\log\left|\frac{\sqrt{2}x-3}{\sqrt{2}x+3}\right| + c$

23. $\int \sqrt{x^2-2}dx$

$= \int \sqrt{x^2-\left(\sqrt{2}\right)^2}dx$

$= \frac{x}{2}\sqrt{x^2-2} - \frac{2}{2}\log\left|x+\sqrt{x^2-2}\right| + c$

$\left[\because \int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$= \frac{x}{2}\sqrt{x^2-2} - \log\left|x+\sqrt{x^2-2}\right| + c$

24. $\int \sqrt{2x^2+4x+1}dx$

$= \int \sqrt{2\left(x^2+2x+\frac{1}{2}\right)}dx$

$= \sqrt{2}\int \sqrt{(x+1)^2-\left(\frac{1}{\sqrt{2}}\right)^2}dx$

$= \sqrt{2}[\frac{x+1}{2}\sqrt{(x+1)^2-\left(\frac{1}{\sqrt{2}}\right)^2}dx$

$= \frac{1/2}{2}\log\left|\sqrt{(x+1)^2-\left(\frac{1}{\sqrt{2}}\right)^2}dx\right| + c$

$\left[\because \int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$= \sqrt{2}\left[\frac{x+1}{2}\sqrt{x^2+2x+\frac{1}{2}} - \frac{1}{4}\log\left|(x+1)+\sqrt{x^2+2x+\frac{1}{2}}\right|\right] + c$

$= \frac{x+1}{\sqrt{2}}\sqrt{2x^2+4x+1} - \frac{\sqrt{2}}{4}\log\left|\sqrt{2}(x+1)+\sqrt{2x^2+4x+1}\right| + c$

25. $\Gamma\left(\frac{7}{2}\right) = \frac{5}{2}\Gamma\left(\frac{5}{2}\right)$
$= \frac{5}{2}\frac{3}{2}\Gamma\left(\frac{3}{2}\right)$
$= \frac{5}{2}\frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right)$
$= \frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi} = \frac{15}{8}\sqrt{\pi}$

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